Coupled Compressor-Diffuser Flow Instability

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The effect on axial compressor instability of the coupling between the compressor and a downstream component (such as a diffuser) is studied analytically and experimentally. It is shown that the presence of an exit diffuser can have an adverse effect on the point of inception of instability, i.e., can move the stall point to a higher mass flow than occurs without the diffuser. Conversely, an exit nozzle, or a downstream compressor operating away from stall, can exert a stabilizing influence. The analytical results are found to be in agreement with experiments carried out on a three-stage axial compressor.

Nomenclature

C_θ = circumferential velocity component
C_x = axial velocity component
D = ratio of static pressure perturbation to total pressure perturbation at compressor exit
h_j = annulus height of jth section of diffuser (see Fig. 2)
i = $\sqrt{-1}$
ℓ = compressor/diffuser axial spacing
ℓ_j = axial length of jth section of diffuser (see Fig. 2)
L = overall diffuser length
n = harmonic number of perturbation Fourier component
p = static pressure
P = total pressure
ΔP = compressor pressure rise
$\Delta P_{T/S}$ = inlet total pressure to exit static pressure compressor pressure rise
R = mean radius of compressor and diffuser
t = time
x = axial coordinate
θ = circumferential coordinate
ρ = density
σ = frequency
σ_i = imaginary part of σ (growth rate)
ψ = perturbation stream function
ω = perturbation vorticity

Subscripts†
j = refers to jth section of diffuser, i.e., p_0 is p in the "0th" section
exit = at compressor exit
inlet = at compressor inlet

Superscripts
$(\bar{\ })$ = mean (unperturbed) flow quantity
$(\ )'$ = perturbation flow quantity

Introduction

As an axial flow compressor is throttled from its design point, the essentially steady, axisymmetric flow that exists becomes unstable. The eventual result of this instability, whether it be rotating stall or surge, is generally a severe degradation in the performance of the compressor, often accompanied by large cyclic stresses to the rotor and stator blades, which both are subjected to a violently unsteady flowfield. As a result of these adverse consequences associated with the instability of the flow, there has been a substantial effort to investigate the conditions under which the flow in an axial compressor becomes unstable, in the sense that an axisymmetric flowfield can no longer exist.

The first of these investigations was by Emmons, Pearson, and Grant,[1] who examined the conditions under which a flow disturbance that was propagating along an infinite cascade would grow. Since then, other investigators have extended the analytical treatments to include the influence of such things as multiple blade rows, a finite lag time in the response of a cascade to changes in the leading-edge conditions, three-dimensional effects, nonlinearity, etc. Two recent useful treatments of this type of problem, which include bibliographies of earlier work, are by Stenning[2] and by Nenni and Ludwig.[3]

It appears, however, that previous investigations have not examined the effect of a coupling between the compressor and other components in the compression system. In other words, the exit boundary condition in the theories in general has been either constant pressure assumed at the exit of the blade row (or last blade row in a multirow configuration), or else a constant area annulus downstream of the compressor with constant static pressure far downstream. In both of these cases, therefore, the compressor is regarded as being isolated from any other component. Under these circumstances, several investigators have shown that the condition for the flowfield to become unstable occurs at the point of zero slope of the inlet total to exit static pressure rise compressor characteristic. This criterion has been found to agree reasonably well with experimental results.[2,4]

In many cases, however, compressors are *not* isolated from their surroundings. For example, it is quite common to run a compressor with an exit diffuser behind it in order to reduce the downstream system losses. It has been shown by Greitzer and Griswold[5] that in this situation there can be a strong coupling between the compressor and the diffuser, as far as the response to a circumferentially nonuniform flow is

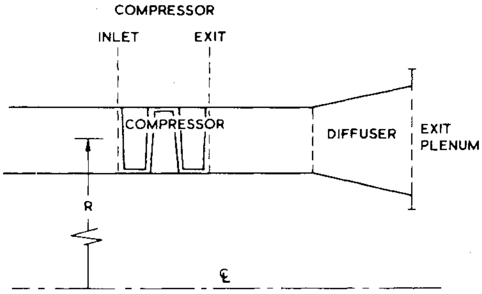

Fig. 1 Compressor/diffuser configuration.

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†No subscript denotes flow variables in region upstream of the compressor.

concerned. Since the instability of the axisymmetric flow (with, for example, the consequent development of a rotating stall cell) can be regarded as the growth of a circumferentially nonuniform perturbation, the possiblity therefore exists that a similar sort of strong coupling also can occur in this latter situation.

If this is so, it is expected that there would be a modification of the conditions at which the instability occurred. Thus, as a result of the presence of the downstream component, the previously mentioned zero slope criterion would be altered, and consequently there would be a change in the flow rate at which instability is encountered in a given compressor.

In order to examine this situation, this paper presents an analysis of the stability of a system consisting of a multistage compressor, followed by another component downstream. The conditions under which a circumferentially nonuniform flow perturbation in this system will grow (leading to the breakdown of the axisymmetric compressor flowfield) are derived. Numerical results then are given in order to show the magnitudes of the effects that are to be expected for values of the relevant physical parameters that are representative of those encountered in turbomachinery applications. Finally, the results are compared with experiments carried out with a three-stage compressor.

Fluid Dynamic Model

The situation to be studied is shown in Fig. 1, in which we see an axial compressor, which is followed by a annular diffuser, which in turn exhausts into a large plenum or collector. The annular flowfield to be studied will be taken as two-dimensional, in that the compressor and annular diffuser are unwrapped at a mean radius, R. Hence, the analysis will be most appropriate to compressor/diffuser configurations of high hub/tip radius ratio. In addition, the fluid is considered to be incompressible and inviscid outside the compressor blading, with the effects of boundary-layer blockage on the diffuser performance being accounted for by use of the effective area ratio concept presented by Greitzer and Griswold.[5] The model that is adopted for the performance of a compressor in a nonuniform flow is that developed by Plourde and Stenning.[6] This assumes that the influence of the very small axial gaps between blade rows in the compressor can be neglected, that the compressor responds in a quasisteady manner to local flow nonuniformities, and that the last stator maintains a constant leaving angle. The overall utility of these assumptions has been demonstrated for flow nonuniformities, which are characterized by circumferential length scales that are much larger than either the blade chord or the axial gap.[2] Since we will be interested primarily in sinusoidal-type perturbations of low orders, this will be the case in the present situation.

In order to investigate the stability of the flow through the configuration shown in Fig. 1, we subject the flowfield to a small perturbation and examine whether it grows or decays, growth being associated with the onset of instability. Since we consider small disturbances, we can adopt a linearized approach and represent a given disturbance as the sum of its Fourier components; and, therefore, one need only examine the response to a single component. In particular, let us choose a disturbance of the form $\exp[i(n\theta-\sigma t)]$, with n real. This is a perturbation that travels around the circumference with phase speed equal to the real part of $\sigma R/n$ and grows or decays according to the sign of the imaginary part of σ. The procedure to be followed is to represent the flow quantities as being composed of a uniform, steady part, plus a perturbation, with the steady part denoted by an overbar and the perturbations by primes. For example, the axial velocity is written

$$C_x=\bar{C}_x+C'_x$$

By use of the equations of motion, the forms of the perturbations can be found in the regions upstream and downstream of the compressor. These then can be related by use of suitable matching conditions across the compressor. The resulting set of equations give rise to an expression for σ, with the value of the imaginary part of σ, σ_i, indicating the stability or instability of the flow. The specific details of the method conveniently can be broken into three parts: the description of the flowfield upstream of the compressor, the description of the flow downstream of the compressor (including within the annular diffuser), and the matching conditions across the compressor.

Consider first the region upstream of the compressor. Since the disturbances that we examine are due to the presence of the compressor, the flow far upstream is unaffected by the perturbations, and therefore is uniform. Consequently, the flow everywhere upstream of the compressor is irrotational, since all of the fluid in this region has convected downstream from uniform flow conditions. Therefore, we can define a stream function for this region which will satisfy Laplace's equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2\psi}{\partial\theta^2}=0 \tag{1}$$

The upstream perturbation velocity components are related to ψ by

$$C'_x=\frac{1}{R}\frac{\partial\psi}{\partial\theta} \tag{2a}$$

$$C'_\theta=-\frac{\partial\psi}{\partial x} \tag{2b}$$

The solution to Eq. (1), satisfying the condition that the disturbance dies away far upstream, is

$$\psi=A\exp[(nx/R)+i(n\theta-\sigma t)] \tag{3}$$

where n is an arbitrary integer, and A is an unknown constant.

Now it is necessary to analyze the time-dependent flow in the region downstream of the compressor. As is commonly the case in practice, this region will be considered to consist of a short section of constant area annulus, followed by an annular diffuser. The flow downstream of the compressor in general is not irrotational because of the vorticity shed by the compressor blades in response to the nonuniform inlet flow.

We are interested primarily in diffusers that are short as compared to the circumference, since this is typical of those diffusers used in turbomachinery applications. In addition, often it is found that multistage compressors of high hup/tip ratio exhibit a one-cell rotating stall behavior, and that generally the most important harmonic of the disturbance will be a low one – i.e., the disturbances of most physical interest are those that have their fundamental wavelength of the order of the circumference of the compressor, and therefore are much longer than the diffuser length. For these reasons, it is adequate to represent the continuous area change of the diffuser by a number of finite changes at discrete locations. In essence we are utilizing a multiactuator disk model of the diffuser. A schematic of the model is shown in Fig. 2. The flow between any two actuator disks is considered to be two-dimensional, and suitable matching conditions are used across the area changes to represent the relevant fluid mechanic effects. Finally, again in keeping with customary design practice, which is nearly axial discharge from the last stage of a compressor, the uniform (unperturbed) flow in the downstream annulus will be taken to be axial.

Let us consider an unrolled annular diffuser, which is made up of sections, as shown in Fig. 2. The flowfield is broken up

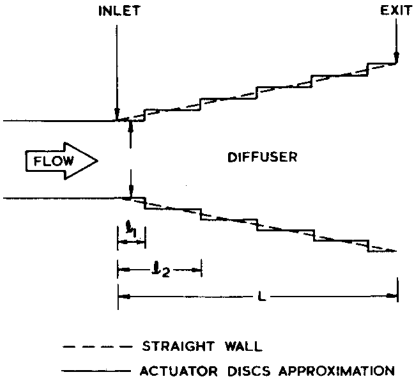

Fig. 2 Discrete area change diffuser representation.

into separate regions, with each having the same mean radius, but a different annulus height h_j. In each region the two-dimensional flow equations apply with the mean velocity depending of course on the local value of the annulus height. The perturbation velocity components in region j are related to the stream function in region j by

$$C'_{x_j} = (1/R)(\partial\psi_j/\partial\theta) \tag{4a}$$

$$C'_{\theta_j} = -\partial\psi_j/\partial x \tag{4b}$$

The vorticity in this region is given by

$$-\nabla^2\psi_j = \omega'_j \tag{5}$$

where the vorticity obeys the equation

$$(\partial\omega'_j/\partial t) + \bar{C}_{x_j}(\partial\omega'_j/\partial x) = 0 \tag{6}$$

The perturbations in all of the different segments of the diffuser must have the same circumferential and temporal dependence, which also must be the same as that in the region upstream of the compressor. Thus we can take ψ and ω' to be of the form

$$\psi_j, \omega'_j \sim \exp[i(n\theta - \sigma t)] \tag{7}$$

Substituting for ω'_j in the vorticity transport equation, Eq. (6) leads to

$$\omega'_j = J_j \exp[i(\sigma x/\bar{C}_{x_j} + n\theta - \sigma t)] \tag{8}$$

where J_j is a constant that must be determined by use of the relevant matching conditions across the discrete area changes.

The stream function can be written as the sum of an irrotational part, which satisfies Laplace's equation, and a rotational part, this latter being the particular solution of Eq. (5). Thus we can express ψ_j as

$$\psi_j = [(A_j \sinh(nx/R) + B_j \cosh(nx/R) + N_j \exp(i\sigma x/\bar{C}_{x_j})] \exp[i(n\theta - \sigma t)] \tag{9}$$

Note that the forms of ω'_j and ψ_j will depend on the value of the uniform axial velocity in the jth section, and are thus valid only in that individual section. In order to find the overall flow pattern we must ensure that the flow quantities are properly matched across the discrete area changes, which are the boundaries of these individual sections. The physical conditions necessary for this are: 1) the mass flow is continuous; 2) the circumferential velocity is continuous (since there is no circumferential force exerted across the area change); and 3) the total pressure is continuous.

The first two of these requirements lead directly to the following matching conditions on the perturbation flow variables at a location $x = \ell_j$ between the jth and the $j+1$th regions

$$h_j C'_{x_j} = h_{j+1} C'_{x_{j+1}} \tag{10a}$$

$$C'_{\theta_j} = C'_{\theta_{j+1}} \tag{10b}$$

It is more convenient to express the third condition in terms of the vorticity than in terms of the total pressure. This can be done by use of the incompressible form of Crocco's theorem to relate the vorticity to the circumferential derivative of the total pressure perturbation, the latter quantity being continuous across the area change. If this is done, the third condition can be written as

$$\omega'_j = \omega'_{j+1}(h_j/h_{j+1}) \tag{10c}$$

In physical terms, this last condition can be viewed as a form of Helmholtz's theorem, which requires that when the radial vortex filaments are stretched, as in going through an increase in annulus height, the vorticity must increase. The three matching conditions [Eq. (10)] apply across each discrete area change, the flow variables that appear being, of course, those that apply on either side of the particular change under consideration.

Using Eqs. (4) and (5), these conditions now can be written in terms of the stream functions ψ_j and ψ_{j+1} as

$$\left.\begin{aligned} \psi_j &= (h_{j+1}/h_j)\psi_{j+1} && (11a)\\ \partial\psi_j/\partial x &= \partial\psi_{j+1}/\partial x && (11b)\\ \nabla^2\psi_j &= \nabla^2\psi_{j+1}(h_j/h_{j+1}) && (11c)\end{aligned}\right\} \text{ at each } x = \ell_j$$

where the quantities ℓ_j denote the values of x at which there are area changes.

Conceptually, now we are able to write down the forms of the velocity perturbations in the flowfield downstream as well as upstream of the compressor. However, at present there are more unknown constants than there are equations, because we have not yet considered the conditions necessary to match the perturbations upstream and downstream of the compressor; it is this aspect that we will now examine.

The specific conditions relating the flow quantities at the front and back ends of the compressor can be stated in the following manner: 1) the axial velocity is continuous across the compressor (C_x at inlet $= C_x$ at exit); 2) the velocity is axial at the *exit* of the compressor; and 3) the pressure rise due to the compressor, which is given by the quasisteady relation between local axial velocity and pressure rise, and therefore is determined by the shape of the uniform flow compressor performance curve, is equal to the difference between the pressures in the fluid at inlet and exit.

Although the compressor is of finite length, the only feature that the axial length contributes is a phase shift (in the θ direction) between the inlet and exit disturbances, and this is of no importance as far as the present calculation is concerned. Therefore, for convenience, we can regard the compressor as having zero axial thickness (another way of looking at this is to say that we are merely redefining the origin of the coordinate system that is used for the downstream region). If this is done, the first two of the previous conditions can be expressed in a straightforward manner as

$$\psi(0,\theta,t) = \psi_0(0,\theta,t) \tag{12a}$$

$$(\partial\psi_0/\partial x)(0,\theta,t) = 0 \tag{12b}$$

where ψ_0 is the stream function in the region immediately downstream of the compressor. In Eqs. (12) it is to be understood that ψ and ψ_0 are evaluated on the upstream and downstream sides of the compressor, respectively.

Before deriving the explicit form for the third condition, some background discussion can be given. The basic compressor model, as described for example by Greitzer and Griswold,[5] assumes that the performance of the compressor in response to small perturbations can be linearized about a mean state and only depends on the local value of the axial velocity. Hence, if ΔP is the compressor pressure rise

$$\Delta P = \overline{\Delta P} + \Delta P' = \overline{\Delta P} + (\overline{\mathrm{d}\Delta P/\mathrm{d}C_x})\, C_x' \qquad (13)$$

where the quantity in parentheses is the value of this quantity, evaluated at the mean flow. The most useful measure of compressor performance in the present situation is the exit static pressure minus inlet total pressure characteristic, which is denoted by $\Delta P_{T/s}$. Thus

$$\Delta P'_{T/S} = (\overline{\mathrm{d}\Delta P_{T/S}/\mathrm{d}C_x})\, C_x'$$

The third matching condition, therefore, can be written as

$$(\overline{\mathrm{d}\Delta P_{T/S}/\mathrm{d}C_x})\, C_x' = p_0' - P' \qquad (14)$$

However, whereas the first two matching conditions were purely kinematic, this third relation involves the dynamics of the flow. Hence, the momentum equation must be used to relate the pressures and velocities, so that we can write this condition in terms of a relation between the stream functions upstream and downstream of the compressor.

The θ component of the momentum equation is

$$\frac{\partial C'_\theta}{\partial t} + \bar{C}_x \frac{\partial C'_\theta}{\partial x} + \frac{1}{\rho R}\frac{\partial p'}{\partial \theta} = 0 \qquad (15)$$

In the region upstream of the compressor, we can make use of the fact that the flow is irrotational, and write this as

$$\frac{\partial C'_\theta}{\partial t} + \frac{\bar{C}_x}{R}\frac{\partial C'_x}{\partial \theta} + \frac{1}{\rho R}\frac{\partial p'}{\partial \theta} = 0 \qquad (16)$$

Since the sum of the second and third terms of Eq. (16) is proportional to the total pressure perturbation, we can express the total pressure perturbation in terms of the stream function as

$$\partial^2\psi/\partial t\partial x = (1/\rho R)(\partial P'/\partial\theta) \qquad (17)$$

Downstreamof the compressor we can use Eq. (15) directly to relate ψ_0 and p_0', remembering that the flow at the exit of the compressor is specified to be axial. Therefore, at exit

$$\partial^2\psi/\partial x^2 = (1/\rho R)(\partial p_0'/\partial\theta) \qquad (18)$$

Equations (16) and (18) can be substituted in Eq. (14) to yield the required matching condition

$$\left(\frac{\overline{\mathrm{d}\Delta P_{T/S}}}{\mathrm{d}C_x}\right)\frac{\partial^2\psi}{\partial\theta^2}(0,\theta,t) = \rho R^2 \bar{C}_{x_0}\frac{\partial^2\psi_0}{\partial x^2}(0,\theta,t) - \rho R\frac{\partial^2\psi}{\partial t\partial x}(0,\theta,t) \qquad (19)$$

The final boundary condition for closure of the problem is that the static pressure at the exit of the annular diffuser is constant, since the diffuser is considered to be dumping into a large plenum. It can be shown that this implies

$$\left(\frac{\partial^2\psi_j}{\partial x\partial t} + \bar{C}_{x_j}\frac{\partial^2\psi_j}{\partial x^2} = 0\right)\Bigg|_{\text{at diffuser exit}} \qquad (20)$$

where the subscript j in Eq. (20) has the value associated with that of the section at the diffuser exit.

When all of the boundary and matching conditions are written explicitly, the result is a set of simultaneous linear equations for the unknown constants A, A_0, B_0, N_0, etc. There are $3m+1$ equations, where m is the total number of sections used. In order for these equations to have a nontrivial solution, it is necessary that their coefficient determinant be equal to zero. This requirement leads to an equation for the frequency σ in terms of the other parameters of the problem. In carrying out the calculations, it was found that it was adequate to divide the diffuser into two sections, with the division between the two regions being placed at a value equal to $1/e$ of the overall diffuser length.

The solution of the equation for σ yields the result that the point of instability (i.e., the point at which the imaginary part of σ is equal to zero) will be determined by the following equation.

$$\frac{1}{\rho\bar{C}_x}\left(\frac{\overline{\mathrm{d}\Delta P_{T/S}}}{\mathrm{d}C_x}\right) = -\left(\frac{D}{1+D}\right) \qquad (21)$$

The quantity D is a nondimensional parameter, which expresses the absolute value of the ratio of static pressure to total pressure nonuniformity at the compressor exit because the diffuser is present. The magnitude of D is determined from the analysis of the circumferentially nonuniform flow in the diffuser. For the case of a constant area annulus downstream of the compressor, $D=0$; and Eq. (21) reduces to the well-known zero slope result referred to in the preceding.

It should be noted that, not only the imaginary part, but the real part of σ (i.e., the phase velocity) is equal to zero at the instability point. Because of this, the stability boundary obtained by the foregoing method is the same point as that obtained by considering the location on the compressor characteristic curve at which the amplification of a *steady* inlet distortion becomes infinite. Specifically, the expression given in Ref. 5 for the ratio of exit to inlet total pressure distortion in the case of steady inlet distortion is

$$\frac{(P')_{\text{exit}}}{(P')_{\text{inlet}}} = -\left\{\frac{1/(1+D)}{[D/(1+D)] \frac{1}{\rho\bar{C}_x}\left(\frac{\overline{\mathrm{d}\Delta P_{T/S}}}{\mathrm{d}C_x}\right)}\right\} \qquad (22)$$

We see that the point at which this ratio becomes unbounded is the same as that obtained previously from a general consideration of the stability of the flow. This dual view of the stability boundary has been pointed out by Stenning for the case of the isolated compressor in which the exit static pressure was constant.[2] What we have shown is that it also can be extended to cases in which the exit static pressure

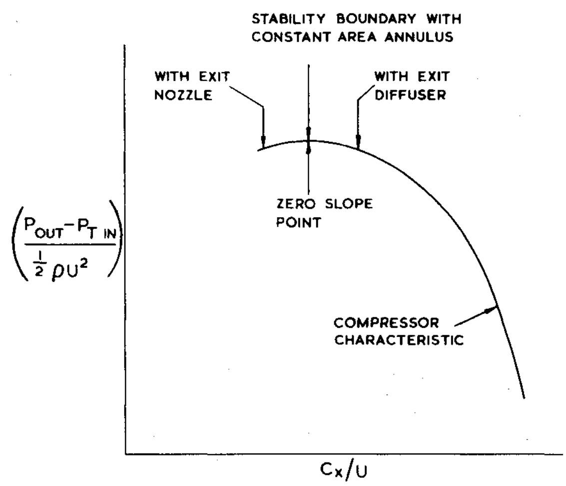

Fig. 3 Effect of exit conditions on compressor stability (schematic).

is definitely not constant – in other words, cases in which the compressor is affected by the presence of a downstream component.

Discussion

The previous analysis indicates that the criterion for the instability of the uniform flow in a compressor/downstream diffuser combination can be obtained by considering the coupled compressor/diffuser response to a steady inlet distortion. The point at which the amplification of the distortion is infinite corresponds to the point of uniform flow instability. This conclusion enables us to make use of the extensive results presented for steady maldistribution in Ref. 5. In addition, it leads readily to a physical understanding of the overall changes in the stability boundary that occur because of the presence of the downstream components. For example, from simple (steady) "diffusers in parallel" arguments (i.e., neglecting the circumferential flow redistribution that actually occurs in asymmetric flow in an annular diffuser‡), one can derive the *qualitative* result that the use of an exit diffuser will move the stability boundary to a point at which the slope of the compressor characteristic is negative.[5] This would occur at a higher flow rate than the zero slope point. Conversely, the arguments also can be applied to the case of "nozzles in parallel." In this latter instance, it is seen readily that the onset of instability will occur at a point of positive slope of the compressor characteristic with a flow rate somewhat lower than that associated with zero slope. Thus, the situation qualitatively will be as sketched in Fig. 3. It should be stressed, however, that the *quantitative* changes in the stability boundary only can be calculated by use of an analysis such as that presented in the preceding, in which the effect of the circumferential velocity components is included.

Quantitative results for an annular diffuser are presented in the next two figures. The effect on the critical slope of the compressor characteristic for a range of annular diffusers is shown in Fig. 4. The perturbation is taken to be a one-cell-per circumference type. The horizontal axis shows the diffuser area ratio parameter, AR-1, and the different curves correspond to different nondimensional lengths. It should be noted that it is the shorter diffusers, in which the effects are greater, that are more characteristic of those used in gas turbine compressor applications. It can be seen that the addition of an exit diffuser can have a significant effect on the slope of the compressor characteristic at which instability is encountered.

These results are for the case of an exit diffuser that is immediately behind the compressor. The influence of the downstream spacing between compressor and diffuser is examined in Fig. 5, using a diffuser that produces a significant effect at zero spacing. The three curves are based on disturbances having one, two, and four lobes. The critical value of the slope vs axial distance between the compressor exit and the diffuser shows a rapid drop in the coupling, so that the two are essentially decoupled if the axial spacing is on the order of a diameter. Further, the higher-order disturbances can be seen both to have less effect at zero spacing and to decay much more rapidly, so that by far the strongest influence comes from the one-per-circumference type of disturbance. The reason for this is that the relevant length scale for interaction is based on the disturbance wavelength. Decreasing this wavelength has the same effect as increasing the diffuser length; it already has been shown that increased difuser length will decrease the influence on the compressor

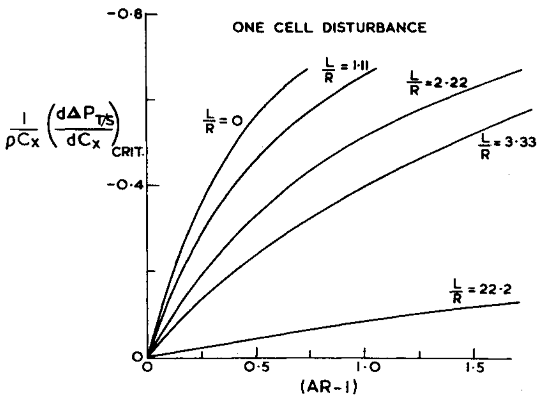

Fig. 4 Effect of exit diffuser on critical slope for instability.

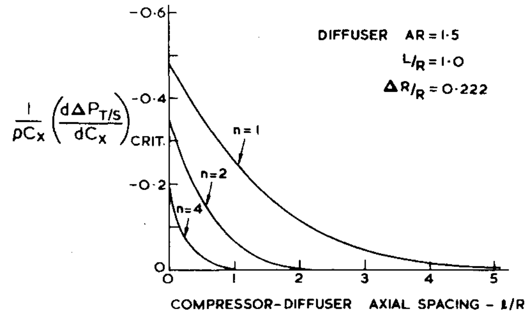

Fig. 5 Effect of spacing on instability boundary.

stability point. In view of the decreased coupling for higher harmonics, the results for the one-per-circumference type of disturbance provide a conservative guide as to the deterioration in stall margin.

Two points should be emphasized in connection with these results. First, the examples have been worked out on the basis of *full span* stall occurring, since this often is observed in compressors of high hub/tip radius ratio, where this analysis is focused. If *part span* stall occurs, the stability point is likely to be affected very little by the presence of the diffuser, since the opportunity of radial flows will decrease the interaction substantially.

Second, the criteria developed here predict a shift from the instability point that is observed with the compressor acting as an isolated compressor, where it is known that the point of zero slope of the total-to-static-pressure-rise characteristic is reasonably adequate for predicting the instability point. However, effects such as unsteady blade response, finite axial gaps, etc., which can act to limit the preciseness of the zero slope prediction, also will be operative in the present cases. In these respects, the present analysis embodies essentially the same limitations as the analyses developed by various authors for the isolated compressor problem.

The concepts developed for use in analyzing the problem of the instability of a coupled compressor-diffuser system also can be applied to other compressor-component interactions. An example concerns operation of a two spool engine, where the two different compressors can be operating at different points on their characteristic, so that while one may be on the verge of stall, the other can be far from stall. Under these conditions it is possible that a (downstream) compressor, which was operating near its design point, say, could exert a favorable effect on a compressor that was operating upstream of it and was matched so as to be near the stall point. By use of the ideas developed in the preceding, this configuration can be analyzed, and it is indeed found that the downstream compressor can stabilize the upstream one. Basically what happens is that the presence of the downstream compressor acts to reduce (attenuate) any axial velocity perturbations, thereby inhibiting the growth of the sort of perturbations that are associated with the onset of a rotating stall cell.

‡Physically, this corresponds to the (somewhat artificial) limiting case where we consider that there are a great many axial vanes, or splitters, placed in the annular diffuser such that there would be many channels "in parallel." Each channel might have a different value of inlet velocity, corresponding to the local value at that circumferential location, but all would have the same area ratio.

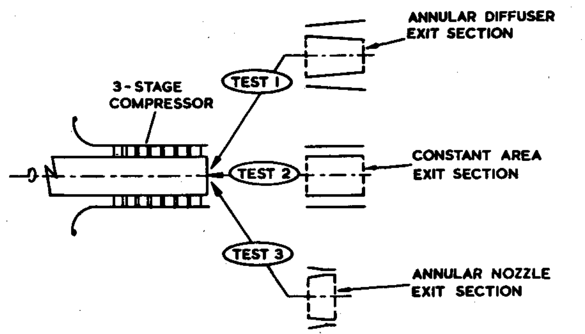

Fig. 6 Schematic of compressor/component interaction experiment.

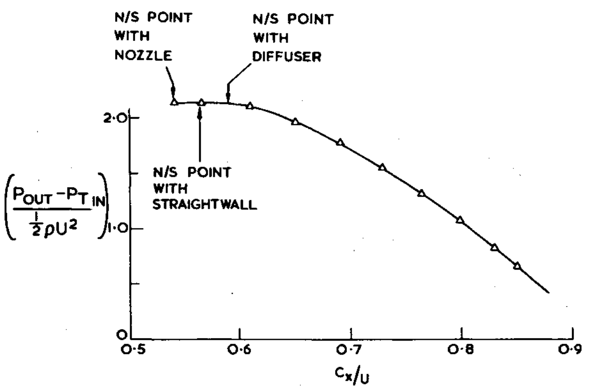

Fig. 7 Effect of exit conditions on compressor stability boundary.

Experimental Results

Experiments have been carried out to examine the overall conclusions of the preceding analysis, as part of a Pratt & Whitney Aircraft program to investigate compressor/component interaction. The compressor used was a low-speed, three-stage research compressor of high hub/tip radius ratio (0.88). With the experimental facility and data acquisition system in use, the experimental uncertainty in both mass flow and pressure-rise measurements was approximately 1%.

Three different exit sections were designed for the three-stage compressor. These were an exit diffuser, which was representative of diffusers used in gas turbine engines, an exit section of constant area, and an exit nozzle. The sections could be attached downstream of the compressor without requiring rebuilding of the compressor itself. The exit sections discharged into a plenum (as in the analytical model) with the flow rate controlled by a throttle on the far downstream end of the plenum. A schematic of the test configurations is shown in Fig. 6.

The exit sections that were used were designed to be positioned far enough downstream of the compressor so that there was no upstream influence of an axisymmetric sort, i.e., no purely radial effects. Thus, changes in stall point are to be associated with an effect having only to do with circumferential nonuniformities considered in the theoretical section.

The experimental procedure conceptually was quite straightforward. The compressor was first run with a diffuser, then with the constant area section, and finally with the nozzle exit section. For each configuration the point of inception of instability (rotating stall) was noted. The results in Fig. 7 show the near stall (N/S) points for the three different exit sections indicated on the compressor characteristic, measured with the exit nozzle. It can be seen that a significant difference exists between the stall points for the three configurations; although away from stall, the performance was found to be essentially the same. The range from the stall point with the diffuser to that with the nozzle is almost 10% in flow. In terms of relative stall margin, defined as the distance between the operating point of the compressor and the stall limit, the differences are considerably larger.

In interpreting the results in the light of the theory, it is to be remembered that the analysis predicts the critical slope at which instability will be encountered. Although it is quite difficult to measure slopes of compressor curves with the accuracy for a precise quantitative comparison with theory, several points can be noted in Fig. 7. The constant area annulus instability point can be seen to occur in the region of the slope of the total-to-static-pressure compressor characteristic equal to zero. Although the measured differences are small, the data also indicate that the stability boundary occurs before the peak in the exit diffuser case. Conversely, in the exit nozzle case, the inception of instability does appear to occur at a point slightly past the peak in the characteristic, so that there has been operation on the positively sloping side of the characteristic. The theory, therefore, does predict the sort of behavior that is observed as far as the shift in the stall point due to exit conditions.

Summary and Conclusions

1) An analysis has been developed to examine the instability of a coupled system consisting of a compressor and a downstream component.

2) It is found that the presence of a nonconstant area annulus downstream of a compressor can exert a significant influence on the point of compressor instability.

3) An exit diffuser tends to be destabilizing, i.e., to shift the instability point to a higher mass flow than that observed with a constant area annulus, whereas an exit nozzle can exert a stabilizing influence.

4) The diffuser (or nozzle) axial length to radius ratio has an important effect on the strength of the interaction, with much stronger coupling occurring for short components, such as those that are typical in turbomachinery applications. Similarly, there is a rapid attenuation of the effect as the axial spacing of the components is increased, with the coupling becoming negligible for distances of a diameter and more.

5) The coupling is strongest for one-per-circumference (single cell) disturbances, and decreases rapidly for higher harmonics.

6) Experimental results obtained with a three-stage compressor confirm the basic theoretical predictions.

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